O K L A H O M A   S T A T E   U N I V E R S I T Y

S C H O O L   O F   E L E C T R I C A L   A N D   C O M P U T E R   E N G I N E E R I N G
S C H O O L O F M E C H A N I C A L A N D A E R O S P A C E E N G I N E E R I N G

ECEN 4413/MAE 4053
Automatic Control Systems
Spring 2007
Final Exam

Choose any four out of five problems.
Please specify which four listed below to be graded:
1)_____; 2)_____; 3)_____; 4)_____;

Name : ______________________________

E-Mail Address:_______________________________________

Problem 1:

The differential equation of a linear system is described by

$$\frac{d^3y(t)}{dt^3} + 5\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 10y(t) = r(t)$$

where $y(t)$ is the output and $r(t)$ is the input.

(a) Draw a state diagram for the system.

(b) Write the state equation from the state diagram. Define the state variables from right to left in ascending order.

(c) Find the characteristic equation and its roots.

(d) Find the transfer function $Y(s)/R(s)$

(e) Find the steady-state output when input is a unit step function, $r(t) = u_s(t)$.

Problem 2:

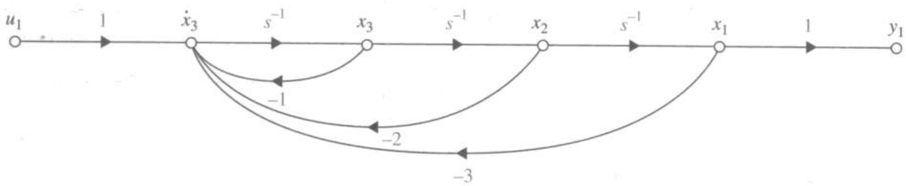
(a) Show that the input-output transfer functions of the two systems shown below are the same.

(b) Write the state-space equations of the system in Fig(a) as

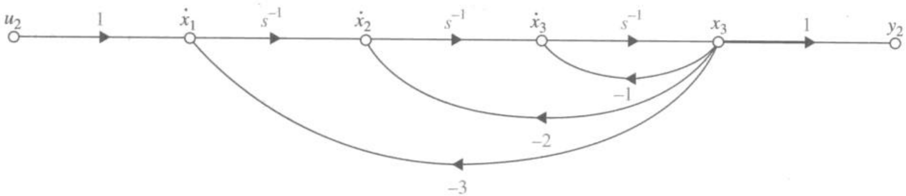
$$\frac{dx(t)}{dt} = A_1 x(t) + B_1 u_1(t), \quad y_1(t) = C_1 x(t)$$

and those of the system in Fig(b) as

$$\frac{dx(t)}{dt} = A_2 x(t) + B_2 u_2(t), \quad y_2(t) = C_2 x(t).$$



(a)



(b)

Problem 3:

Given a nonlinear system described by

$$\ddot{y} - \dot{y} - e^{a+1} y = 3\ddot{u} + 4\dot{u} + 2u \,,$$

linearize the system about the equilibrium point and show the linearized state space representation in $\dot{x} = Ax + Bu,\ y = Cx + Du$.

Problem 4:

Given the dynamic equations of a time-invariant system

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t), \quad y(t) = Cx(t)$$

where $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}.$

Find the matrices A_1 and B_1 so that the state equations for the same system are written as

$$\frac{d\bar{x}(t)}{dt} = A_1\bar{x}(t) + B_1u(t)$$

where $\bar{x}(t) = \begin{bmatrix} x_1(t) \\ y(t) \\ dy(t)/dt \end{bmatrix}.$

Problem 5:

A linear time-invariant system is described by the following state equation

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t)$$

where $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

The closed-loop system is implemented by state feedback, i.e., $u(t) = -Kx(t)$, where $K = [k_1 \quad k_2 \quad k_3]$; and k_1, k_2 and k_3 are real constants. Determine the constraints on the elements of K so that the closed-loop system is stable.